Laws of motion

(circular motion)

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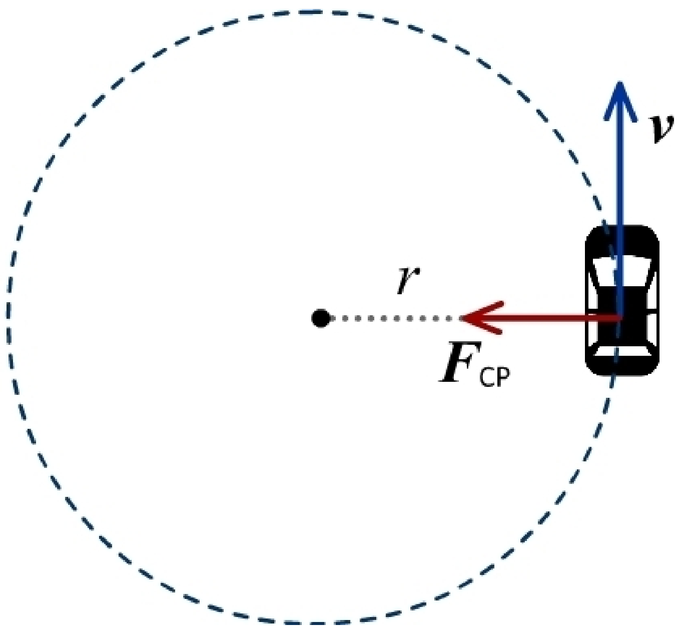
Note : The notes given in this file is no substitute to the much detailed discussion held in the online/contact classes with active participation of students. It , at best, serves the purpose of ready reference for important concepts/derivations covered in the classes.

Maximum speed for safe turn

Consider a car of mass m moving on a circular track of radius r. The centripetal force to turn along the track is provided by the frictional force between the tyres and the ground. Therefore

$$\frac{mv^2}{r} = F_S$$



As the static frictional force is a self adjusting in nature, the centripetal force required should be less than or equal to the limiting frictional force. Therefore

$$\frac{m\,v^2}{r} \leq \mu\, m\, g$$

$$v^2 \leq \mu\, r\, g$$

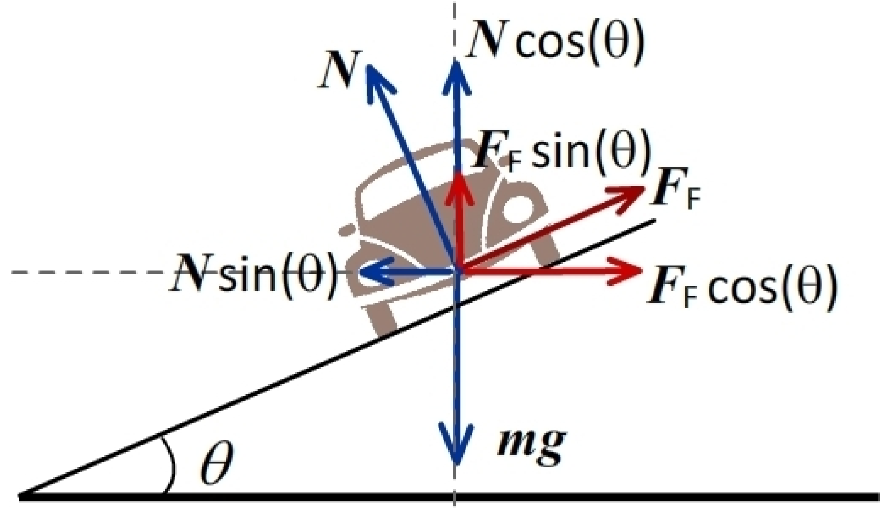
$$v \leq \sqrt{\mu\, r\, g}$$

The car will skid on the circular track if the speed exceeds the limit given by the above relation.

Therefore the maximum velocity for a safe turning on a flat road is $\sqrt{\mu\, r\, g}$

Banked road : Minimum speed

Consider a car of mass M moving on a circular banked track of radius r. Let θ be the angle of banking. The centripetal force to turn along the track is provided by the horizontal component of normal reaction.



Considering vertical component,

$$N\cos(\theta)+F_F\sin(\theta)=mg$$

$$N\cos(\theta)+\mu N\sin(\theta)=mg$$

$$\Rightarrow \quad N=\frac{mg}{(\cos(\theta)+\mu\sin(\theta))} \quad \boxed{\text{i}}$$

Considering horizontal component

$$N\sin(\theta)-F_F\cos(\theta)=\frac{mv^2}{r}$$

$$N\sin(\theta)-\mu N\cos(\theta)=\frac{mv^2}{r}$$

$$N(\sin(\theta)-\mu\cos(\theta))=\frac{mv^2}{r}$$

Using the value of N from eq (i) we get

$$\frac{mg(\sin(\theta)-\mu\cos(\theta))}{(\cos(\theta)+\mu\sin(\theta))}=\frac{mv^2}{r}$$

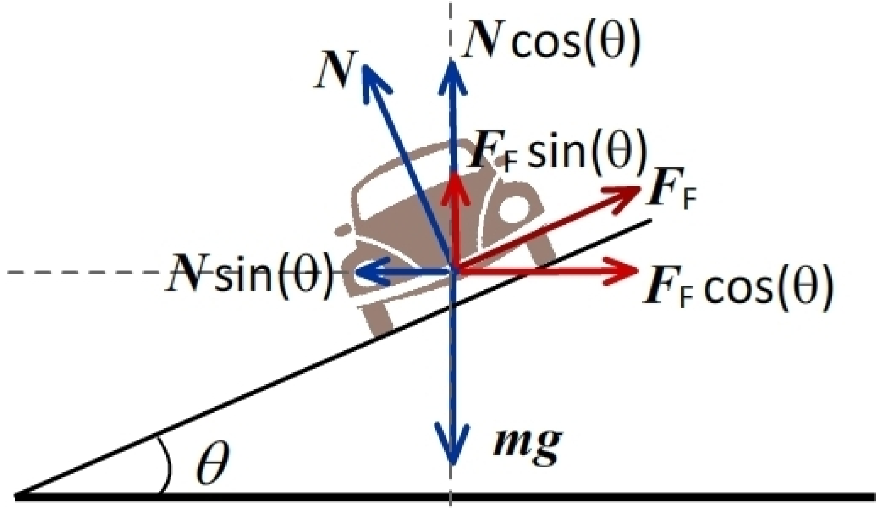
Banked road : Minimum speed

$$\frac{g\left(\sin(\theta) - \mu\cos(\theta)\right)}{\left(\cos(\theta) + \mu\sin(\theta)\right)} = \frac{v^2}{r}$$

$$v^2 = \frac{rg\left(\tan(\theta) - \mu\right)}{1 + \mu\tan(\theta)}$$

$$v_{\text{min}} = \sqrt{\frac{rg\left(\tan(\theta) - \mu\right)}{1 + \mu\tan(\theta)}}$$

This is the minimum speed of safe turning around the curve.



- Should a vehicle always be moving (and not be stationary) on a banked road?
- If it can remain stationary, then why and what is this minimum velocity?
- What if μ is less than tan(θ) !

Laws of motion

Banked road : Maximum speed

For the maximum possible speed of safe turn (without skidding outwards) the frictional force acts inwards. Therefore

Considering vertical component,

$$N\cos(\theta) = mg + F_F\sin(\theta)$$

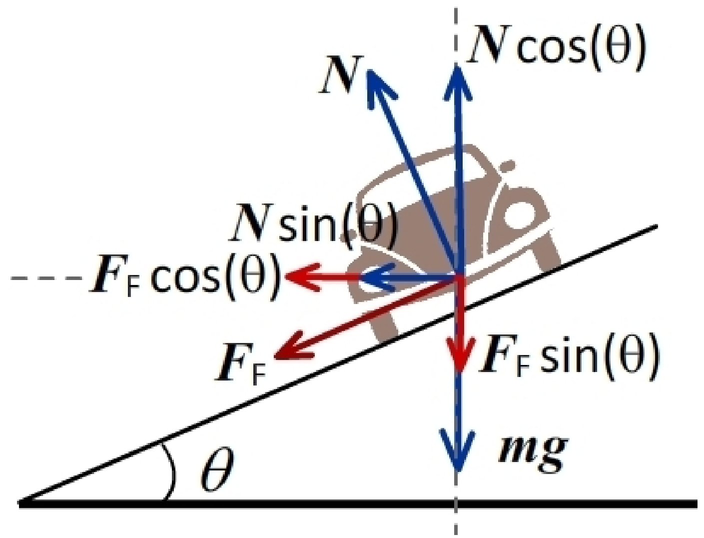
$$N\cos(\theta) - \mu N\sin(\theta) = mg$$

$$\Rightarrow \quad N = \frac{mg}{(\cos(\theta) - \mu\sin(\theta))} \qquad \boxed{i}$$

Considering horizontal component

$$N\sin(\theta) + F_F\cos(\theta) = \frac{mv^2}{r}$$

$$N\sin(\theta) + \mu N\cos(\theta) = \frac{mv^2}{r}$$



$$N(\sin(\theta) + \mu\cos(\theta)) = \frac{mv^2}{r}$$

Using the value of N from eq (i) we get

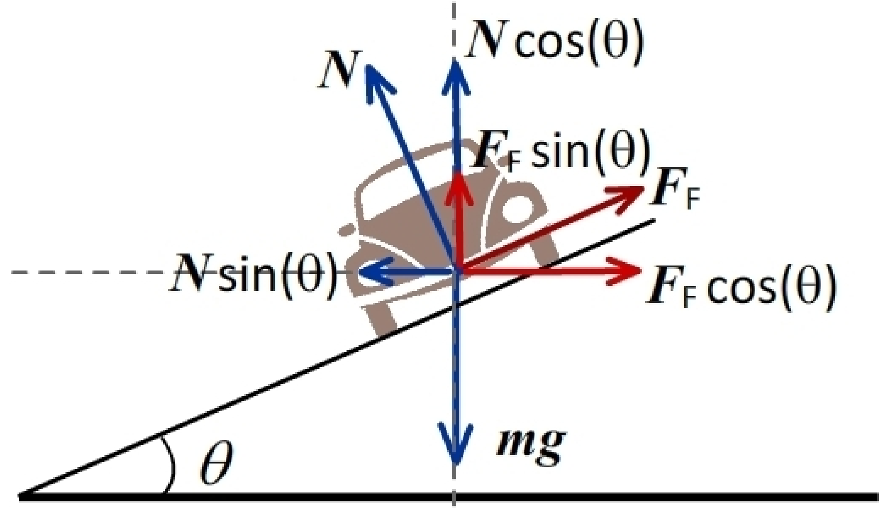
$$\frac{mg(\sin(\theta) + \mu\cos(\theta))}{(\cos(\theta) - \mu\sin(\theta))} = \frac{mv^2}{r}$$

Banked road : Maximum speed

$$\frac{g(\sin(\theta)+\mu\cos(\theta))}{(\cos(\theta)-\mu\sin(\theta))}=\frac{v^2}{r}$$

$$v^2=\frac{rg(\tan(\theta)+\mu)}{1-\mu\tan(\theta)}$$

$$v_{\max}=\sqrt{\frac{rg(\tan(\theta)+\mu)}{1-\mu\tan(\theta)}}$$

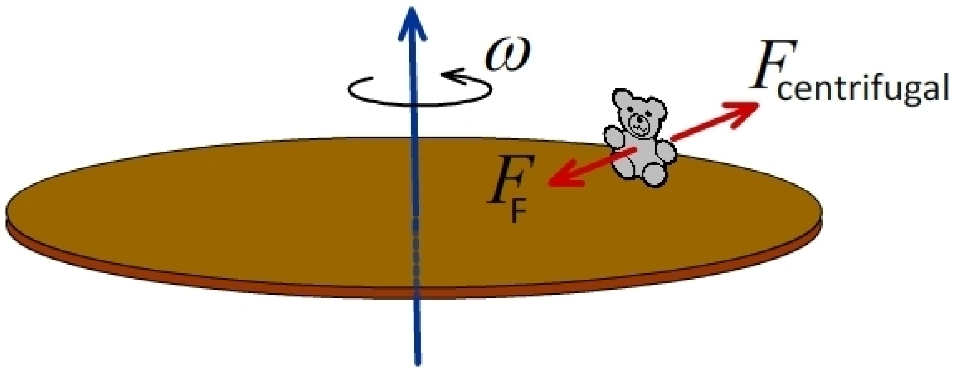


This is the maximum speed of safe turning around the curve.

Briefly explain centrifugal force with suitable examples.

Centrifugal force is the pseudo force. It is used as a correction term when a body is observed from a revolving/rotating frame of reference.
Centrifugal force acts along the radius and away from the center of the circular path.

$$F_{\text{centrifugal}} = m\,r\,\omega^2$$



Consider a rough horizontal disc rotation at an angular speed ω. Let a body, placed on the disc at a distance of r from its centre be at rest w.r.t the disc. For an observer on the disc the body appears to be stationary therefore using Newton's laws the FBD would include a pseudo force acting outwards as shown below.

For an observer on ground, the body moves along the circular path due to frictional force and pseudo-force correction (centrifugal force) is not required.

Application : A centrifuge is a device that is used for separating heavier particles and light particles.